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RESEARCH MEMORANDUM

ANALYSIS OF EFFICIENCY CHARACTERISTICS OF A SINGLE-STAGE TURBINE WITH DOWNSTREAM STATORS IN TERMS OF WORK AND SPEED REQUIREMENTS

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NATIONAL ADVISORY COMMITTEE
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SUMMARY

The efficiency characteristics of a single-stage turbine with downstream stators are analyzed. It is assumed that the mean-section flow is one-dimensional and that the blade specific losses are proportional to the average specific kinetic energy. The efficiencies are presented in terms of a work-speed parameter λ, defined as the ratio of the square of the mean-section blade speed to the required specific work output. Range of this parameter of 0 to 0.5 is that applicable in critical turbojet engines as well as in turbopumps and accessory drives. The efficiencies investigated are total or aerodynamic, rating, and static. Impulse and maximum-efficiency velocity diagrams are considered. The efficiencies of these two types are compared with that of the single-stage impulse velocity diagram, since this case yielded higher rating and static efficiencies for the reaction limit imposed on the rotor. The downstream stator loading is limited by a suction-surface diffusion factor of 0.5.

Below λ of 0.38 the rating and static efficiencies can be increased by the addition of downstream stators. With one stator, the maximum-efficiency velocity diagram reaches the diffusion limit of 0.5 first for zero turbine exit whirl at λ of 0.29, whereas the impulse velocity diagram reaches this limit at λ of 0.205. By the addition of a second downstream stator, the impulse case reaches limiting diffusion at λ of 0.095. After the diffusion limit is reached, it is necessary to impose progressively more turbine exit whirl in order not to exceed the diffusion limit as λ is reduced. With one downstream stator, up to about 16 points increase in rating efficiency and $12\frac{1}{2}$ points increase in static efficiency can be obtained, depending on the region of λ. With two downstream stators, about 24 points increase in rating efficiency and 20 points increase in static efficiency are possible, depending on the region of λ.

INTRODUCTION

Analytical studies to determine the effect of variations in work and speed requirements on over-all turbine efficiency are currently underway

at the NACA Lewis laboratory. The fundamental parameter λ used is "Parson's characteristic number" (ref. 1), which is defined as the ratio of the square of the mean-section blade speed to the specific work output for design conditions.

In reference 2, single-stage turbine efficiency was determined for a range of the work-speed parameter λ from 0 to 1.0, which is applicable to jet engines as well as turbopumps and accessory drives. Results of this analysis indicate that, in the range of λ from 0 to 0.5, turbine efficiencies are considerably decreased as a result of (1) increased viscous losses due to the increased flow velocities required for a given work output and (2) high turbine exit kinetic energy required to prevent negative reaction across the turbine. Most of this kinetic energy is in terms of exit whirl.

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One means of improving the efficiency of turbines in the lower range of work-speed parameter (λ of 0 to 0.5) is using stators downstream of the turbine. These stators turn the flow out of the rotor back toward the axial direction and thus reduce the whirl losses out of the turbine. Net improvement in efficiency can be realized, if the reduction in exit-whirl losses more than compensates for the additional viscous losses due to the added stators.

This analysis evaluates the effect of adding downstream stators on turbine efficiencies based on the work-speed parameter λ. The range of λ from 0 to 0.5 covered in the analysis is for critical jet-engine applications and for turbopumps and accessory drives, where low to moderate blade speeds are used with high specific work outputs. The single-stage turbine with downstream stators of this analysis is hereinafter referred to as a "$1\frac{1}{2}$-stage turbine."

As in reference 2, three types of efficiency are considered:

(1) Total, or aerodynamic, efficiency, which includes all aerodynamic losses

(2) Rating efficiency, which, in addition to aerodynamic losses, considers turbine exit whirl a loss (used in jet-engine analysis)

(3) Static efficiency, which, in addition to aerodynamic losses, considers turbine exit total velocity head a loss (used for turbopumps and turbine accessory drives)

These efficiencies are presented for impulse and maximum-efficiency mean-section velocity diagrams.

METHOD OF ANALYSIS

Efficiency Equations

The total, rating, and static efficiencies are developed in terms of the work-speed parameter λ and the upstream-stator exit-whirl parameter $V_{u,1}/\Delta V_u$, which determines the velocity diagram to be used. (All symbols are defined in appendix A.) The parameter λ is defined

$$\lambda = \frac{U^2}{gJ\Delta h'} \tag{1}$$

where $\Delta h'$ is the actual specific work output. The following equations for efficiencies are developed for turbines with one downstream stator.

Total efficiency. - The total efficiency includes all aerodynamic losses:

$$\eta = \frac{\Delta h'}{\Delta h'_{id}} \tag{2}$$

where $\Delta h'_{id}$ is the ideal specific work output corresponding to the total-pressure ratio across the turbine. The expression $(\Delta h'_{id} - \Delta h')$ is the loss in specific energy across the turbine and is introduced into equation (2) as

$$\eta = \frac{\Delta h'}{\Delta h' + (\Delta h'_{id} - \Delta h')} \tag{3}$$

Using the general equation relating the specific work output, blade speed, and change in absolute whirl velocity, and solving for U give

$$U = \frac{gJ\Delta h'}{\Delta V_u} \tag{4}$$

Substituting equation (4) into equation (1) for U and solving for $\Delta h'$ yield

$$\Delta h' = \frac{\lambda(\Delta V_u)^2}{gJ} \tag{5a}$$

Combining equations (1) and (4) also yields

$$\lambda = \frac{U}{\Delta V_u} \tag{5b}$$

Equation (5a) is substituted into equation (3) for $\Delta h'$ and is divided through by $(\Delta V_u)^2/gJ$ to obtain

$$\eta = \frac{\lambda}{\lambda + \frac{gJ(\Delta h'_{id} - \Delta h')}{(\Delta V_u)^2}} \tag{6}$$

The turbine specific energy loss $(\Delta h'_{id} - \Delta h')$ is assumed equal to the sum of the rotor and stator blade specific losses $(L_{S,1} + L_R + L_{S,2})$. 4175
The blade specific loss is defined as the difference between the ideal and the actual specific kinetic energy obtained by expanding to the blade exit static pressure. This assumption is analyzed in appendix C of reference 2 for a single-stage turbine. By a similar analysis, including the downstream-stator loss, it could be shown that the sum of the rotor and stator specific losses is approximately equal to the turbine specific-energy loss except for a small effect of a change in absolute enthalpy level through the turbine. As a result of this assumption, equation (6) can be modified to

$$\eta = \frac{\lambda}{\lambda + \frac{gJ(L_{S,1} + L_R + L_{S,2})}{(\Delta V_u)^2}} \tag{7}$$

If it is assumed, as in reference 2, that the sum of the blade specific losses in the turbine is proportional to the product of the ratio of turbine blade surface area to weight flow and the average specific kinetic-energy level E of the turbine, then

$$(L_{S,1} + L_R + L_{S,2}) \sim \frac{A}{w} E \tag{8}$$

In order that the velocity-diagram characteristics will be compatible with the loss assumptions used herein, the specific kinetic-energy level E of the turbine is represented by the average of the specific kinetic energies entering and leaving the blade rows where the velocities are relative to the blade rows. A typical set of $1\frac{1}{2}$-stage turbine velocity diagrams is shown in figure 1. The specific kinetic energy of the stator is

$$\frac{V_{x,0}^2 + V_{x,1}^2 + V_{u,1}^2}{4g}$$

of the rotor,

$$\frac{V_{x,1}^2 + W_{u,1}^2 + V_{x,2}^2 + W_{u,2}^2}{4g}$$

and of the downstream stator,

$$\frac{V_{x,2}^2 + V_{u,2}^2 + V_{x,3}^2 + V_{u,3}^2}{4g}$$

The axial velocity squared assumed is an average of the sum of the individual axial velocities squared:

$$\frac{(V_x^2)_{av}}{2g} = \frac{V_{x,0}^2 + 2V_{x,1}^2 + 2V_{x,2}^2 + V_{x,3}^2}{12g}$$

As a result, the average specific kinetic energy can be written

$$E = \frac{6(V_x^2)_{av} + V_{u,1}^2 + W_{u,1}^2 + W_{u,2}^2 + V_{u,2}^2 + V_{u,3}^2}{12g} \tag{9}$$

The same over-all constant ($B = 0.030$ from ref. 2) for full-admission turbines is used in this analysis:

$$B = K \frac{A}{w} \tag{10}$$

This parameter consists of the ratio of surface area to weight flow A/w and a constant of proportionality K. The constant K includes the loss in kinetic energy per unit surface area in terms of the free-stream specific kinetic energy, which is assumed constant. Included in the denominator of K is a factor of number of blade rows obtained from the averaging of the kinetic energies. When the number of blade rows is increased by the addition of downstream stator blade rows, A/w is correspondingly increased. As a result of these counterbalancing effects, the parameter B is independent of number of blade rows.

Substituting equations (8) to (10) into equation (7) yields

$$\eta = \frac{\lambda}{\lambda + B\left[\frac{6(V_x^2)_{av} + V_{u,1}^2 + W_{u,1}^2 + W_{u,2}^2 + V_{u,2}^2 + V_{u,3}^2}{(\Delta V_u)^2}\right]} \tag{11}$$

Using the equations relating the absolute and relative tangential velocities of the rotor and equation (5b),

$$V_{u,2} = V_{u,1} - \Delta V_u$$

$$W_{u,1} = V_{u,1} - U$$

$$W_{u,2} = V_{u,1} - U - \Delta V_u$$

and substituting into equation (11) give the equation for total efficiency:

$$\eta = \frac{\lambda}{\lambda + B\left[6\lambda\frac{(V_x^2)_{av}}{gJ\Delta h'} + \left(\frac{V_{u,1}}{\Delta V_u}\right)^2 + \left(\frac{V_{u,1}}{\Delta V_u} - \lambda\right)^2 + \left(\frac{V_{u,1}}{\Delta V_u} - \lambda - 1\right)^2 + \left(\frac{V_{u,1}}{\Delta V_u} - 1\right)^2 + \left(\frac{V_{u,3}}{\Delta V_u}\right)^2\right]} \tag{12}$$

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Rating efficiency. - In a jet engine the tangential component of kinetic energy out of the turbine does not add to engine thrust and therefore is considered a loss chargeable to the turbine. The rating efficiency of the turbine is defined

$$\eta_x = \frac{\Delta h'}{\Delta h'_{id,x}} \tag{13}$$

In reference 2, for a single-stage turbine, the rating efficiency is developed to be

$$\eta_x = \frac{\eta}{1 + \frac{V_{u,2}^2}{2gJ\Delta h'_{id}}} \tag{14}$$

In this analysis, the whirl out of the downstream stator is the loss chargeable to the turbine; equation (14) is therefore changed to

$$\eta_x = \frac{\eta}{1 + \frac{V_{u,3}^2}{2gJ\Delta h'_{id}}} \tag{15}$$

Using equations (2) and (5a) in equation (15) modifies it to

$$\eta_x = \frac{\eta}{1 + \frac{\eta}{2\lambda}\left(\frac{V_{u,3}}{\Delta V_u}\right)^2} \tag{16}$$

Static efficiency. - In many applications such as turbopumps and auxiliary drives, the entire kinetic energy leaving the turbine is considered lost; therefore, the performance is based on the total- to static-pressure ratio across the turbine. The static efficiency of the turbine is

$$\eta_s = \frac{\Delta h'}{\Delta h'_{id,s}} \tag{17}$$

By the same method used in developing the rating efficiency, the following equation is obtained for static efficiency:

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$$\eta_s = \frac{\eta}{1 + \frac{\eta}{2}\left[\frac{1}{\lambda}\left(\frac{V_{u,3}}{\Delta V_u}\right)^2 + \frac{V_{x,3}^2}{gJ\Delta h'}\right]} \tag{18}$$

The exit axial velocity squared is assumed equal to the average axial velocity squared, and equation (18) is changed to

$$\eta_s = \frac{\eta}{1 + \frac{\eta}{2}\left[\frac{1}{\lambda}\left(\frac{V_{u,3}}{\Delta V_u}\right)^2 + \frac{(V_x^2)_{av}}{gJ\Delta h'}\right]} \tag{19}$$

The axial component of velocity usually increases through the turbine; and, therefore, the static efficiencies presented herein may be somewhat higher than those obtained when the actual axial velocity is used. The value of 0.49 for the parameter $(V_x^2)_{av}/gJ\Delta h'$ falls in the range currently encountered in turbine designs at the Lewis laboratory (see ref. 2).

Additional downstream stators. - In the very low range of work-speed parameter, the efficiencies decrease rapidly owing to the limited flow turning in one downstream stator. It is possible to obtain an increase in rating and static efficiencies by the use of additional downstream stators to turn the flow back to the axial direction. The method of developing turbine-efficiency equations including the additional downstream stators is the same as for one downstream stator. The following equations of total, rating, and static efficiencies are for two downstream stators:

$$\eta = \frac{\lambda}{\lambda + B\left[8\lambda\frac{(V_x^2)_{av}}{gJ\Delta h'} + \left(\frac{V_{u,1}}{\Delta V_u}\right)^2 + \left(\frac{V_{u,1}}{\Delta V_u} - \lambda\right)^2 + \left(\frac{V_{u,1}}{\Delta V_u} - \lambda - 1\right)^2 + \left(\frac{V_{u,1}}{\Delta V_u} - 1\right)^2 + 2\left(\frac{V_{u,3}}{\Delta V_u}\right)^2 + \left(\frac{V_{u,4}}{\Delta V_u}\right)^2\right]} \tag{20}$$

$$\eta_x = \frac{\eta}{1 + \frac{\eta}{2\lambda}\left(\frac{V_{u,4}}{\Delta V_u}\right)^2} \tag{21}$$

$$\eta_s = \frac{\eta}{1 + \frac{\eta}{2}\left[\frac{1}{\lambda}\left(\frac{V_{u,4}}{\Delta V_u}\right)^2 + \frac{(V_x^2)_{av}}{gJ\Delta h'}\right]} \tag{22}$$

Velocity-Diagram Considerations

Two types of velocity diagram can be used for a specified value of λ. The velocity diagrams are changed by varying the upstream-stator exit-whirl parameter $V_{u,1}/\Delta V_u$.

Impulse conditions. - For the purpose of this analysis, impulse conditions are defined by equal relative velocities into and out of the rotor. Assuming the axial components of velocity into and out of the rotor are equal,

$$W_{u,1} = -W_{u,2}$$

Since

$$\Delta V_u = W_{u,1} - W_{u,2}$$

and

$$V_{u,1} = W_{u,1} + U$$

the preceding three equations are combined to form the following relation (derived in ref. 2):

$$\frac{V_{u,1}}{\Delta V_u} = \lambda + \frac{1}{2} \tag{23}$$

When equation (23) is substituted into equation (12), the following equation of total efficiency for impulse conditions results:

$$\eta = \frac{\lambda}{\lambda + B\left[6\lambda\frac{(V_x^2)_{av}}{gJ\Delta h'} + 2\lambda^2 + 1 + \left(\frac{V_{u,3}}{\Delta V_u}\right)^2\right]} \tag{24}$$

Maximum total efficiency. - There is a maximum value of total efficiency for a given λ at an upstream-stator exit whirl other than that for impulse conditions (ref. 2). When the quantity within the brackets in equation (12) is a minimum, the total efficiency is a maximum. If $V_{u,3}/\Delta V_u$ is held constant, the partial derivative of the brackets [ ] with respect to $V_{u,1}/\Delta V_u$ is

$$\frac{\partial[\;]}{\partial \frac{V_{u,1}}{\Delta V_u}} = 2\left(\frac{V_{u,1}}{\Delta V_u}\right) + 2\left(\frac{V_{u,1}}{\Delta V_u} - \lambda\right) + 2\left(\frac{V_{u,1}}{\Delta V_u} - \lambda - 1\right) + 2\left(\frac{V_{u,1}}{\Delta V_u} - 1\right)$$

Setting this equation equal to zero and solving for $V_{u,1}/\Delta V_u$ give the following relation:

$$\left(\frac{V_{u,1}}{\Delta V_u}\right)_{\eta_{max}} = \frac{\lambda + 1}{2} \tag{25}$$

Substituting equation (25) into equation (12) then gives the following equation for maximum total efficiency:

$$\eta = \frac{\lambda}{\lambda + B\left[6\lambda \frac{(V_x^2)_{av}}{gJ\Delta h'} + \lambda^2 + 1 + \left(\frac{V_{u,3}}{\Delta V_u}\right)^2\right]} \tag{26}$$

Maximum rating and static efficiencies. - The rating and static efficiencies for a given value of λ also reach a maximum value for a velocity diagram other than that of the impulse case. If the quantity $V_{u,3}/\Delta V_u$ is considered constant for a given λ, the relationship of equation (25) holds for the rating and static efficiencies, as $(V_x^2)_{av}/gJ\Delta h'$ is constant also. Thus,

$$\left(\frac{V_{u,1}}{\Delta V_u}\right)_{\eta_{x,max}} = \frac{\lambda + 1}{2} \tag{27}$$

$$\left(\frac{V_{u,1}}{\Delta V_u}\right)_{\eta_{s,max}} = \frac{\lambda + 1}{2} \tag{28}$$

Velocity diagrams. - Turbine rotor exit characteristics $(V_{u,2}/\Delta V_u = V_{u,1}/\Delta V_u - 1)$ are shown in figure 2 as a function of λ for several types of velocity diagram. A range of λ from 0 to 1.0 is covered in order to present a more complete picture, although the analysis presented herein is concerned only with values of λ from 0 to 0.5. The impulse case representing the straight-line function of the rotor exit-whirl parameter of -0.5 at $\lambda = 0$ to 0.5 at $\lambda = 1.0$ is the borderline between positive and negative rotor reaction. The negative-reaction region is crosshatched to represent an undesirable region of rotor operation and is not considered in this analysis. Single-stage-rotor exit-whirl characteristics from reference 2 are shown for comparison. The curve shown for $1\frac{1}{2}$-stage maximum efficiency represents total, rating, and static efficiencies.

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Downstream-Stator Diffusion

By the use of a downstream stator, higher rotor exit whirl can be specified than is normally used in conventional single-stage turbines without the associated high losses. For the same λ, improved static and rating efficiencies can be produced under the same turbine operating conditions. The downstream-stator losses are kept within the loss assumptions of the analysis by specifying a limit to the stator loading by the suction-surface diffusion across the downstream stator. The diffusion factor of reference 3 is modified in appendix B in terms of the upstream- and downstream-stator exit-whirl parameters $V_{u,1}/\Delta V_u$ and $V_{u,3}/\Delta V_u$, respectively, and work-speed parameter λ to 4175

$$D = \frac{\left[\left(\frac{V_{u,1}}{\Delta V_u} - 1\right)^2 + \lambda \frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2} - \left[\left(\frac{V_{u,3}}{\Delta V_u}\right)^2 + \lambda \frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2} + \frac{1}{2\sigma}\left[\frac{V_{u,3}}{\Delta V_u} - \left(\frac{V_{u,1}}{\Delta V_u} - 1\right)\right]}{\left[\left(\frac{V_{u,1}}{\Delta V_u} - 1\right)^2 + \lambda \frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2}} \tag{B5}$$

The diffusion-factor characteristics are plotted in figure 3 as a function of the work-speed parameter λ for both impulse and maximum-efficiency velocity diagrams. A solidity σ of 1.5 was chosen from reference 3 as the highest value for which information was available. A higher value of solidity would result in a lower diffusion factor for a given λ. The region above a diffusion factor of 0.5 is indicated as undesirable in order to keep the losses within the assumptions of this analysis. Only the curve for zero turbine exit whirl for the maximum-efficiency diagram is shown, since it is advantageous to transfer to the impulse velocity diagram after the diffusion-factor limit is reached for maximum efficiency at $\lambda = 0.29$, as will be shown later. The impulse case with one downstream stator and zero whirl reaches the limiting diffusion factor at a λ of 0.205. At a reduced work-speed parameter where the specified diffusion factor is limiting the loading on the first downstream stator, it may be necessary to go to whirl out of the downstream stator in order not to exceed the limit placed on the diffusion factor in this analysis.

If a second downstream stator is added when the first downstream stator reaches the limiting diffusion factor, the work-speed parameter can be further reduced until the second downstream stator reaches the diffusion-factor limit before going to turbine exit whirl. From appendix B, the equation for the limiting diffusion factor on the second downstream stator is

$$D = \frac{\left[\left(\frac{V_{u,3}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2} - \left[\left(\frac{V_{u,4}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2} + \frac{1}{2\sigma}\left(\frac{V_{u,4}}{\Delta V_u} - \frac{V_{u,3}}{\Delta V_u}\right)}{\left[\left(\frac{V_{u,3}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2}} \tag{B6}$$

In figure 3, the diffusion factor on the second downstream stator is shown only for impulse across the rotor, since, with this case, a lower work-speed parameter can be used for the same diffusion factor than if the maximum-efficiency velocity diagram were used.

Figure 4 is a reproduction of the data of figure 2 showing only the range of λ covered for the velocity diagrams considered in this analysis. When the maximum-efficiency case reaches the limiting diffusion factor, a transition is made at limiting diffusion and zero turbine exit whirl to limiting diffusion of the impulse case. Lines of constant downstream-stator exit whirl are shown. It can be seen that, for a specified work-speed parameter, less downstream-stator exit whirl is required for the impulse case than for the maximum-efficiency case when the diffusion factor is limiting. The turbine exit whirl for the impulse case, reduced by the addition of the second downstream stator, is also shown in figure 3. It is interesting to note that limiting diffusion on the second downstream stator is reached when the exit-whirl parameter of the first downstream stator $V_{u,3}/\Delta V_u$ is -0.2.

The Mach number level through the turbine would have an effect on the losses of the downstream stators when a diffusion limit is specified. Inasmuch as this analysis is to be as general as possible, the diffusion limit was chosen without consideration of Mach number level.

RESULTS OF ANALYSIS

The efficiencies of the velocity-diagram types considered herein are compared with the single-stage impulse case of reference 2, which had higher values of rating and static efficiencies in the range of $\lambda = 0$ to approximately 0.5 without going to negative reaction. Total, rating, and static efficiencies for a given case reach limiting diffusion at the same work-speed parameter for zero turbine exit whirl. With one downstream stator, the maximum-efficiency case reached the limiting diffusion factor at λ of 0.29, and the impulse case reached the limiting diffusion factor at λ of 0.205. By the use of a second downstream stator, the impulse case reached the limiting diffusion factor at λ of 0.095, as shown in the following discussion.

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Total Efficiency

The total-efficiency characteristics are presented in figure 5. The single-stage impulse efficiency is higher than the $1\frac{1}{2}$-stage efficiencies, since the total loss of the turbine is increased by the loss occurring in the added downstream stators. The $1\frac{1}{2}$-stage maximum-efficiency curve is about 1/2 point above the $1\frac{1}{2}$-stage impulse curve and about parallel to it up to a limiting diffusion factor for zero turbine exit whirl on the $1\frac{1}{2}$-stage impulse curve. As the work-speed parameter is reduced, the maximum-efficiency curve shows progressively more downstream-stator exit whirl after it reaches limiting diffusion. It crosses and then falls below the impulse curve. If limiting diffusion were not included, the maximum-efficiency curve would be above the impulse curve over the entire range of λ. 4175

The total efficiency of the impulse case with two downstream stators is the lowest for a given λ because of the losses added by the second stator. This turbine configuration reaches limiting diffusion at λ of 0.095 and a stator exit-whirl parameter $V_{u,3}/\Delta V_u$ of -0.2.

Rating Efficiency

The rating efficiencies are compared in figure 6. From λ of 0.38 to 0, there is an advantage in using a downstream stator. The part of the maximum-efficiency curve for zero downstream-stator exit whirl, where diffusion is not limiting, is fairly flat, varying only 3 points from $\eta_x = 0.86$ at λ of 0.5 to about $\eta_x = 0.83$ at λ of 0.29. For the same conditions for impulse with one downstream stator, the curve varies 5 points from $\eta_x = 0.85$ at λ of 0.5 to $\eta_x = 0.80$ at λ of 0.205. The curves cross at λ of 0.26. After the maximum-efficiency case reaches the limiting diffusion of 0.5, about 1/2 point in efficiency can be gained by using the transition of limiting diffusion and zero downstream-stator exit whirl to the impulse case rather than going directly to the impulse velocity diagram. When both cases reach limiting diffusion, the efficiencies decrease rather sharply as λ is reduced. By adding a downstream stator to a single-stage turbine, it is possible to reach a reduced λ of approximately 0.20 before the efficiency drops below 0.80, as compared with λ of 0.29 for the single-stage turbine.

The turbine rating efficiency can be improved further (below λ of 0.16 and $V_{u,3}/\Delta V_u$ of -0.1) by adding a second downstream stator with the rotor at impulse conditions. After limiting diffusion is reached on both downstream stators, the efficiency decreases rapidly as the work-speed parameter is further reduced.

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Static Efficiency

The static-efficiency characteristics are shown in figure 7. The static-efficiency level is not so high as that of the rating efficiency because of the kinetic energy involved in the exit axial velocity. Below λ of about 0.38, the use of a downstream stator is advantageous. The $1\frac{1}{2}$-stage curves are fairly flat and about 1/2 point apart up to limiting diffusion. The maximum-efficiency curve varies 2 points from $\eta_s = 0.71$ at λ of 0.5 to $\eta_s = 0.69$ at λ of 0.29, and the impulse curve varies $3\frac{1}{2}$ points from $\eta_s = 0.705$ at λ of 0.5 to $\eta_s = 0.67$ at λ of 0.205 for zero downstream-stator exit whirl. As with the rating efficiency, about 1/2 point in static efficiency is to be gained after the maximum-efficiency case reaches limiting diffusion by making the transition to the impulse case at limiting diffusion and zero turbine exit whirl. With the addition of one downstream stator, λ is then reduced from 0.26 to 0.17 before going below a static efficiency of 0.65.

The addition of the second downstream stator improves static efficiency below λ of 0.16 and $V_{u,3}/\Delta V_u$ of -0.1.

Efficiency Comparisons

Figures 8(a) and 9(a) show the possible net gain in rating efficiency by using downstream stators based on a comparison with the single-stage impulse case of reference 2. A gain in efficiency of about 16 points, or about 33 percent, can be obtained, with the maximum at about λ of 0.1, by using the $1\frac{1}{2}$-stage impulse velocity diagram. The maximum-efficiency case is slightly higher down to λ of 0.26; below this value, the impulse case has the higher efficiency. When a second downstream stator is added, an improvement of up to 24 points, or about 75 percent, is possible over the single-stage impulse case for a λ of approximately 0.06.

Figures 8(b) and 9(b) show the gain in static efficiency by using downstream stators as compared with the single-stage impulse case of reference 2. A gain of about $12\frac{1}{2}$ points, or about 35 percent, is possible around λ of 0.075 with one downstream stator for the impulse case. With two downstream stators at a λ of 0.06, about 20 points, or 65 percent, can be gained in efficiency for the impulse case.

It might be pointed out that, from velocity-diagram considerations, a work-speed parameter λ of about 0.1 is the practical lower limit that can be expected in a conventional single-stage turbine with a high specific work output.

SUMMARY OF RESULTS

The efficiency characteristics of a single-stage full-admission turbine with downstream stators have been analyzed as a function of turbine work and speed requirements. The range of the work-speed parameter λ used (from 0 to 0.5) is that in which single-stage turbines with downstream stators ("$1\frac{1}{2}$-stage turbines") offer an efficiency advantage over conventional turbines for critical applications in turbojet engines, turbopumps, and accessory drives. Impulse and maximum-efficiency velocity diagrams have been considered. The efficiencies of these two types were compared with that of the single-stage impulse velocity diagram, which yielded higher rating and static efficiencies in the range of λ considered in this analysis. The loading on the downstream stators was limited by restricting the diffusion factor to 0.5. The following results were obtained:

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1. The $1\frac{1}{2}$-stage turbine with one downstream stator has a distinct advantage in rating and static efficiencies below λ of 0.38. The efficiencies do not vary to any extent until the limiting diffusion factor is reached, after which they decrease rather sharply. Limiting diffusion at zero turbine exit whirl for the maximum-efficiency velocity diagram is reached first at λ of 0.29 and for the impulse case at about λ of 0.205. For the range of λ between the limiting diffusion of both cases, up to about 1/2-point increase in efficiency is obtainable by making a transition between the two velocity-diagram types at zero downstream-stator exit whirl and limiting diffusion. When a second downstream stator is added, an additional increase in rating and static efficiencies over that of one downstream stator can be obtained below λ of 0.16.

2. Rating-efficiency improvements in the impulse case of about 16 points, or 33 percent, are possible with the addition of one downstream stator, the maximum occurring around λ of 0.1. With two downstream stators, improvements in efficiency up to about 24 points, or 75 percent, are possible, the maximum occurring at λ of 0.06.

3. Static-efficiency improvements in the impulse case of about $12\frac{1}{2}$ points, or 35 percent, at λ of 0.075 are possible with the addition of one downstream stator. With two downstream stators, static efficiency is improved about 20 points, or 65 percent, at λ of 0.06 over the single-stage impulse case.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, October 23, 1956

APPENDIX A

SYMBOLS

A	turbine blade surface area, sq ft
B	parameter equal to $K\frac{A}{w}$
D	diffusion factor
E	average specific kinetic-energy level
g	acceleration due to gravity, 32.17 ft/sec^2
$\Delta h'$	specific work output, Btu/lb
J	mechanical equivalent of heat, 778.2 ft-lb/Btu
K	constant of proportionality
L	specific loss in kinetic energy, Btu/lb
m,n	parameters used in developing diffusion factor (appendix B)
U	mean-section blade speed, ft/sec
V	absolute gas velocity, ft/sec
W	relative gas velocity, ft/sec
w	weight-flow rate, lb/sec
η	total efficiency, based on total- to total-pressure ratio across turbine
η_s	static efficiency, based on total- to static-pressure ratio across turbine
η_x	rating efficiency, based on ratio of total pressure upstream of turbine to axial total pressure downstream of turbine
λ	work-speed parameter, $U^2/gJ\Delta h'$
σ	solidity, ratio of chord to spacing

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Subscripts:

av average

id ideal

R rotor

S stator

s static 4175

u tangential

x axial

0 upstream of turbine

1 station between upstream stator and rotor, upstream stator

2 station between rotor and first downstream stator, first downstream stator

3 station downstream of first downstream stator

4 station downstream of second downstream stator when used

APPENDIX B

DEVELOPMENT OF DOWNSTREAM-STATOR DIFFUSION FACTOR IN TERMS OF WORK-SPEED AND STATOR-EXIT-VELOCITY PARAMETERS

In order to place a limit on the losses that may occur in the downstream stator, a limit is placed on the suction-surface diffusion. The basic expression used for this limit (ref. 3) is

$$D = 1 - \frac{V_3}{V_2} + \frac{\Delta V_{u,2-3}}{2\sigma V_2} \tag{B1}$$

This expression is a function of the over-all change in velocity across the stator and a term proportional to the conventional lift coefficient of the section based on the blade inlet velocity. Because of the sign convention used in this analysis, the quantity $\Delta V_{u,2-3}$ is negative. Therefore, using absolute quantities and simplifying equation (B1) give

$$D = 1 - m - \frac{1}{2\sigma n} \tag{B2}$$

where

$$m = \frac{V_3}{V_2} = \frac{\left(V_{u,3}^2 + V_{x,3}^2\right)^{1/2}}{\left(V_{u,2}^2 + V_{x,2}^2\right)^{1/2}}$$

and

$$n = \frac{V_2}{V_{u,2} - V_{u,3}} = \frac{\left(V_{u,2}^2 + V_{x,2}^2\right)^{1/2}}{V_{u,2} - V_{u,3}}$$

after resolving the velocities into their components. In this analysis the axial velocity is assumed constant through the turbine, and the square of the axial velocity at each station is represented by an average of the squares $(V_x^2)_{av}$. Dividing each expression by the change in tangential velocity across the rotor ΔV_u and substituting in the average axial velocity give:

$$m = \frac{\left[\left(\frac{V_{u,3}}{\Delta V_u}\right)^2 + \frac{(V_x^2)_{av}}{(\Delta V_u)^2}\right]^{1/2}}{\left[\left(\frac{V_{u,2}}{\Delta V_u}\right)^2 + \frac{(V_x^2)_{av}}{(\Delta V_u)^2}\right]^{1/2}}$$

$$n = \frac{\left[\left(\frac{V_{u,2}}{\Delta V_u}\right)^2 + \frac{(V_x^2)_{av}}{(\Delta V_u)^2}\right]^{1/2}}{\frac{V_{u,2}}{\Delta V_u} - \frac{V_{u,3}}{\Delta V_u}}$$

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Solving equation (5a) for $(\Delta V_u)^2$ and substituting into expressions m and n in equation (B2) yield

$$D = 1 - \frac{\left[\left(\frac{V_{u,3}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2}}{\left[\left(\frac{V_{u,2}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2}} + \frac{\frac{V_{u,3}}{\Delta V_u} - \frac{V_{u,2}}{\Delta V_u}}{2\sigma\left[\left(\frac{V_{u,2}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2}} \tag{B3}$$

By rearranging terms slightly,

$$D = \frac{\left[\left(\frac{V_{u,2}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2} - \left[\left(\frac{V_{u,3}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2} + \frac{1}{2\sigma}\left(\frac{V_{u,3}}{\Delta V_u} - \frac{V_{u,2}}{\Delta V_u}\right)}{\left[\left(\frac{V_{u,2}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2}} \tag{B4}$$

Substituting $V_{u,1}$ for $V_{u,2}$ in equation (B4) gives

$$D = \frac{\left[\left(\frac{V_{u,1}}{\Delta V_u} - 1\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2} - \left[\left(\frac{V_{u,3}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2} + \frac{1}{2\sigma}\left[\frac{V_{u,3}}{\Delta V_u} - \left(\frac{V_{u,1}}{\Delta V_u} - 1\right)\right]}{\left[\left(\frac{V_{u,1}}{\Delta V_u} - 1\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2}} \tag{B5}$$

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The downstream-stator exit-whirl parameter $V_{u,3}/\Delta V_u$ is solved directly from equation (B5) by substituting specified values and solving the resulting quadratic equation.

The diffusion factor for the second downstream stator is modified from equation (B1) in the same manner as for the first downstream stator to give

$$D = \frac{\left[\left(\frac{V_{u,3}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2} - \left[\left(\frac{V_{u,4}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2} + \frac{1}{2\sigma}\left(\frac{V_{u,4}}{\Delta V_u} - \frac{V_{u,3}}{\Delta V_u}\right)}{\left[\left(\frac{V_{u,3}}{\Delta V_u}\right)^2 + \lambda\frac{(V_x^2)_{av}}{gJ\Delta h'}\right]^{1/2}} \tag{B6}$$

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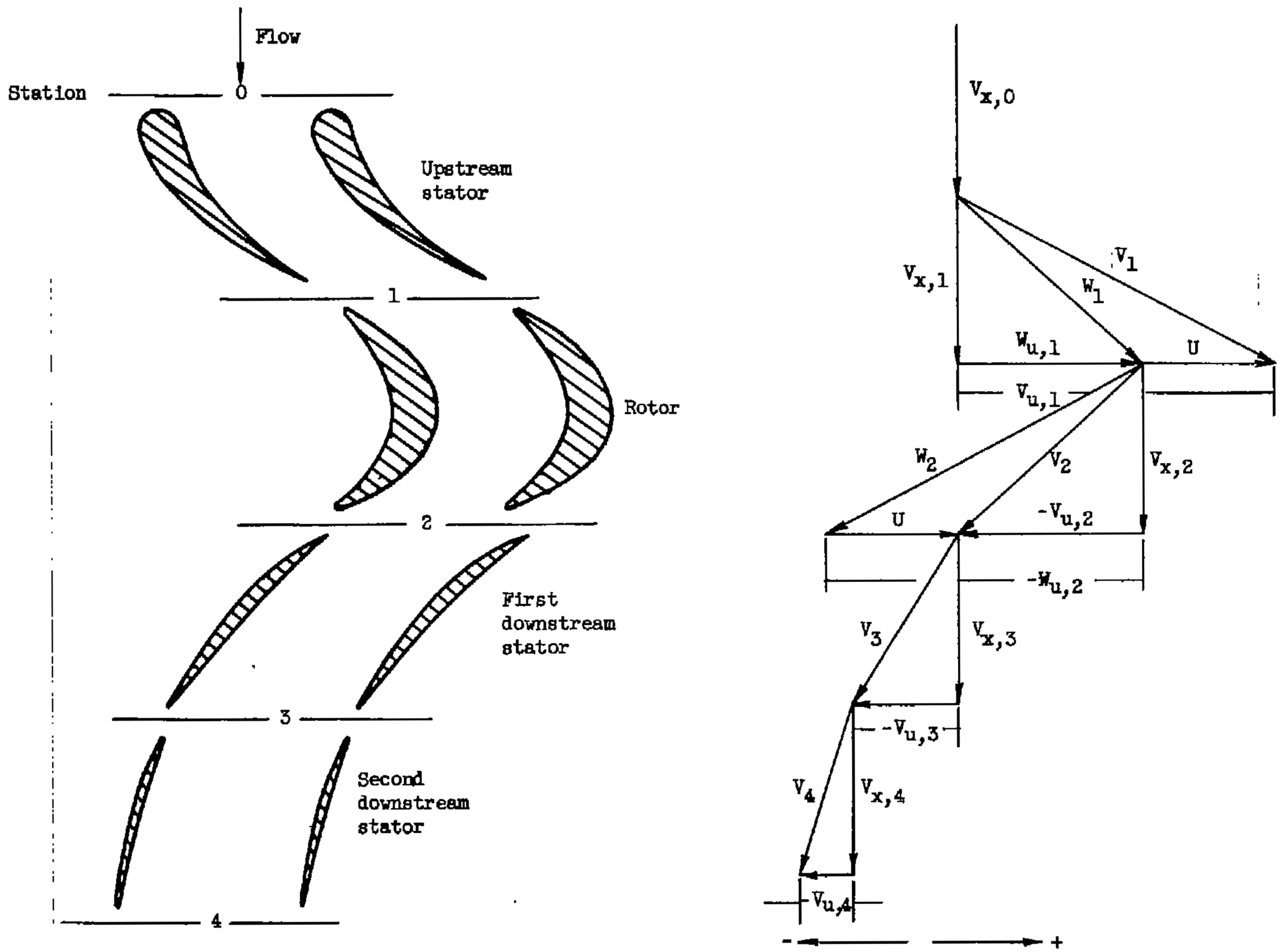

Figure 1. - Typical velocity diagrams.

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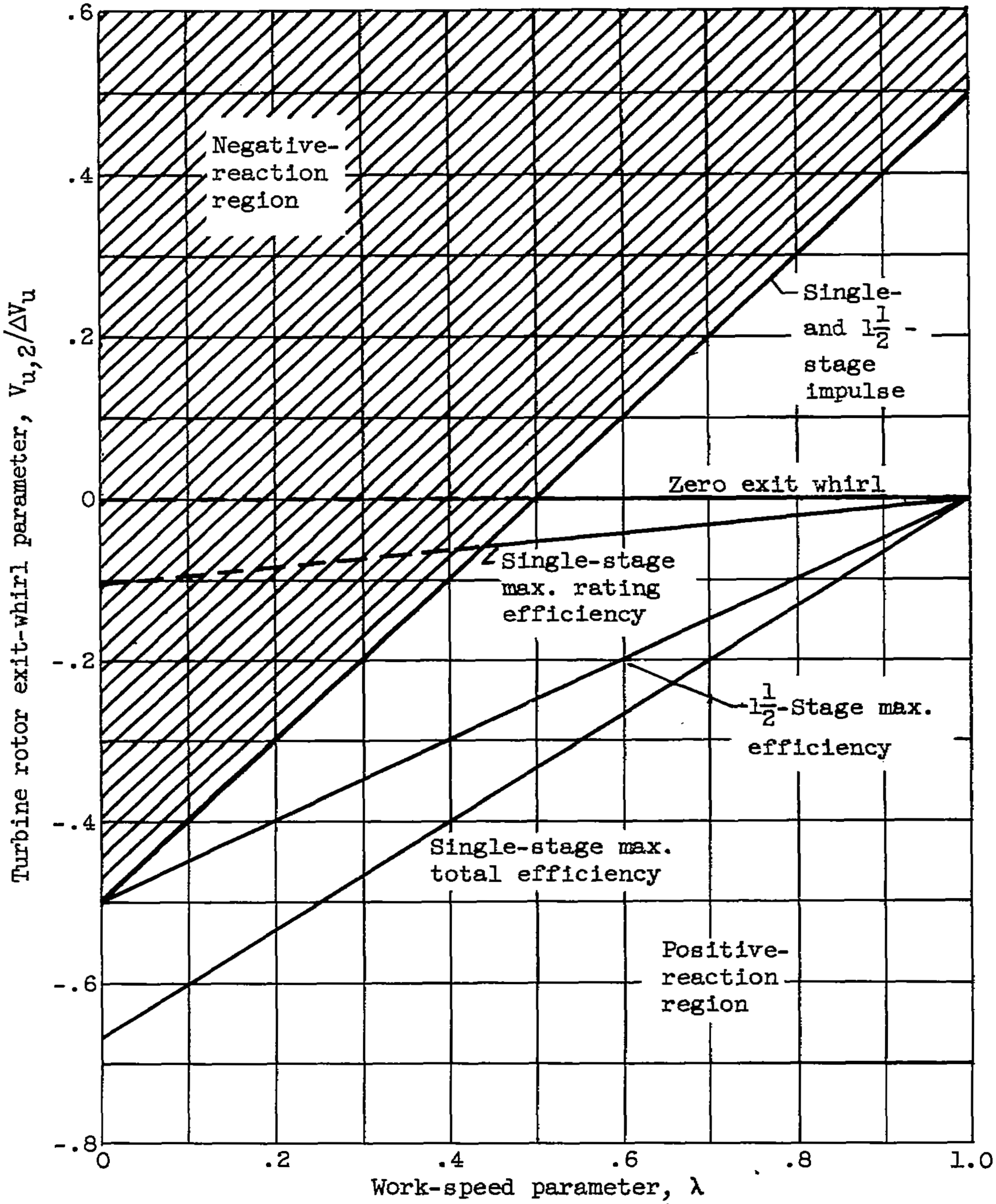

Figure 2. - Turbine rotor exit-whirl characteristics for specified velocity-diagram conditions.

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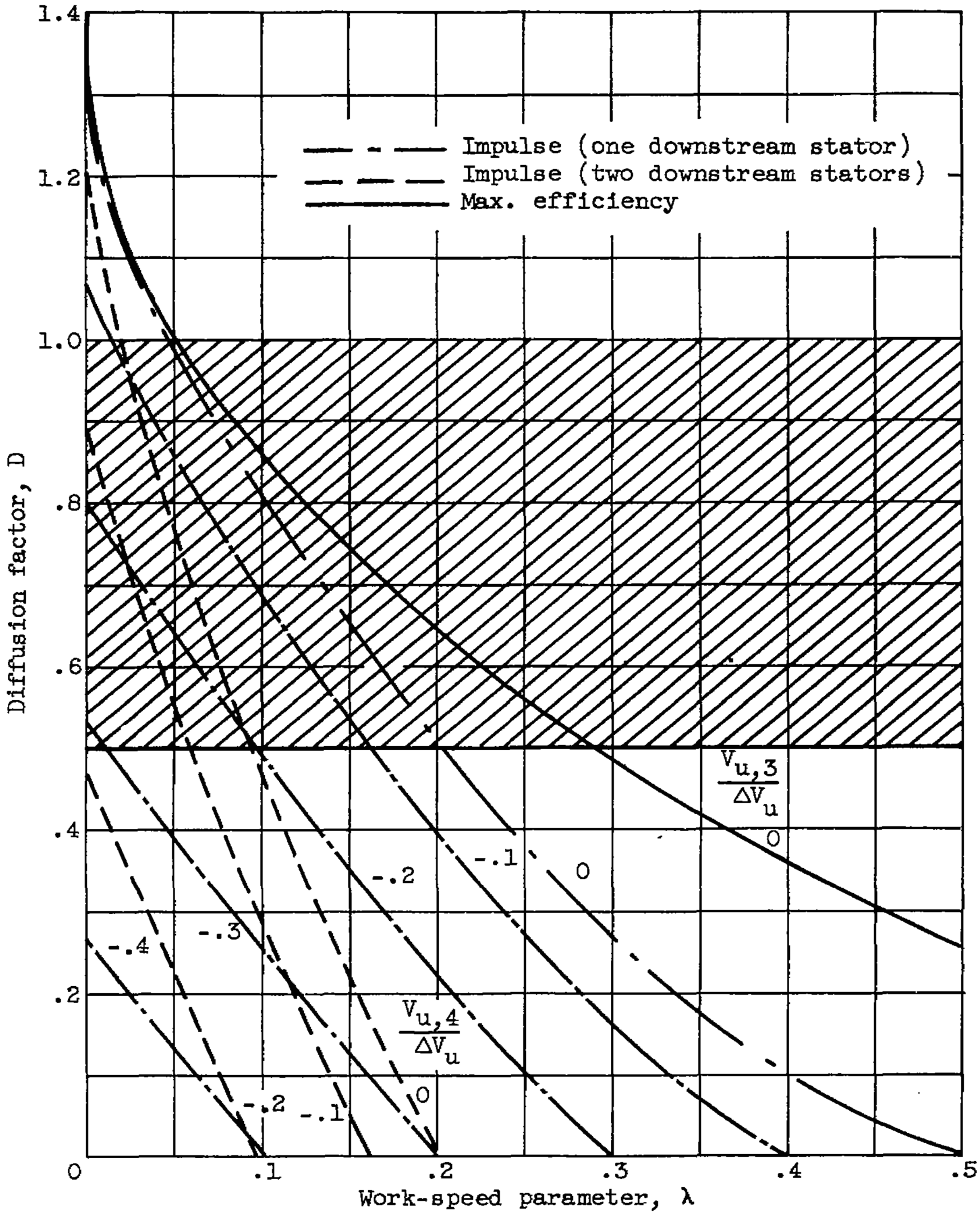

Figure 3. - Downstream-stator diffusion characteristics as function of work-speed parameter for solidity of 1.5.

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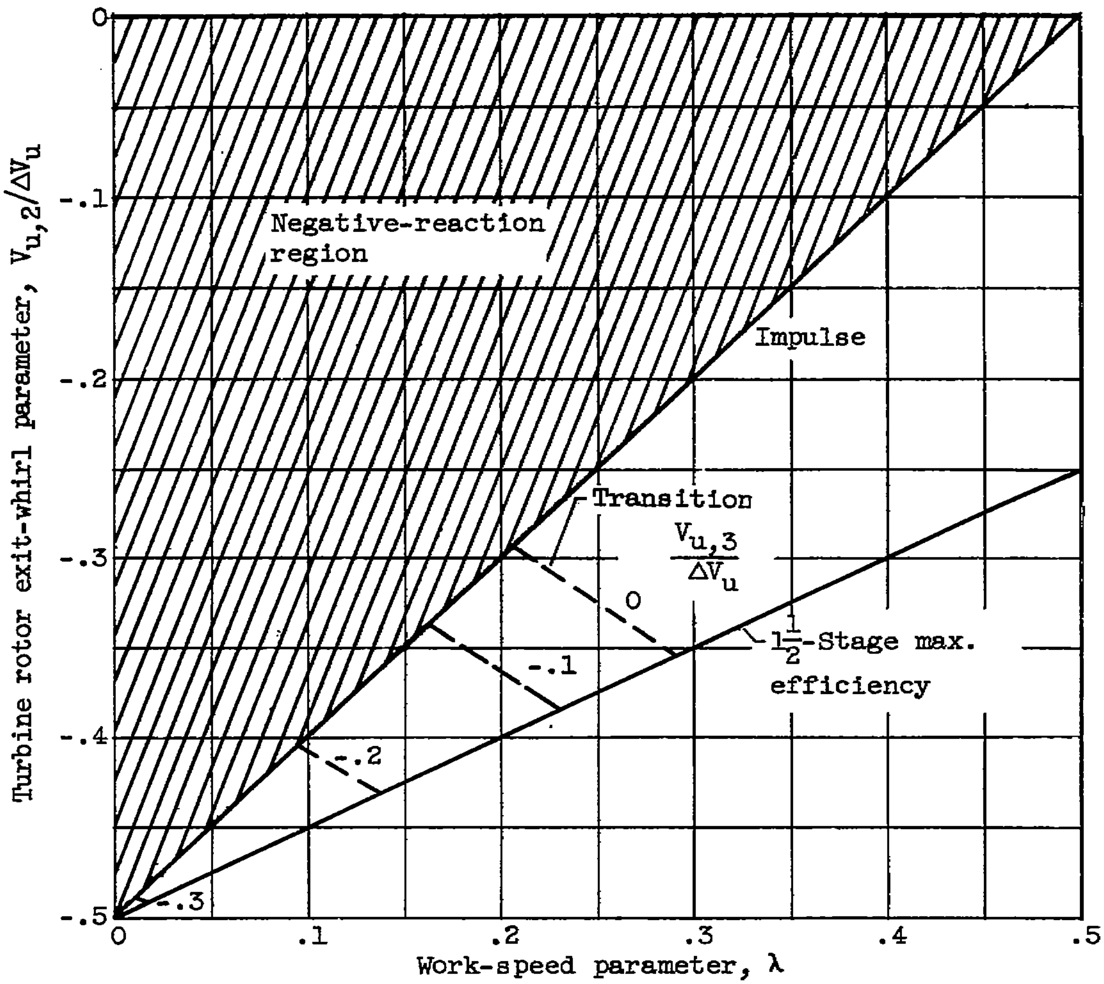

Figure 4. - $1\frac{1}{2}$-Stage turbine rotor exit-whirl characteristics for specified velocity-diagram conditions and diffusion factor of 0.5.

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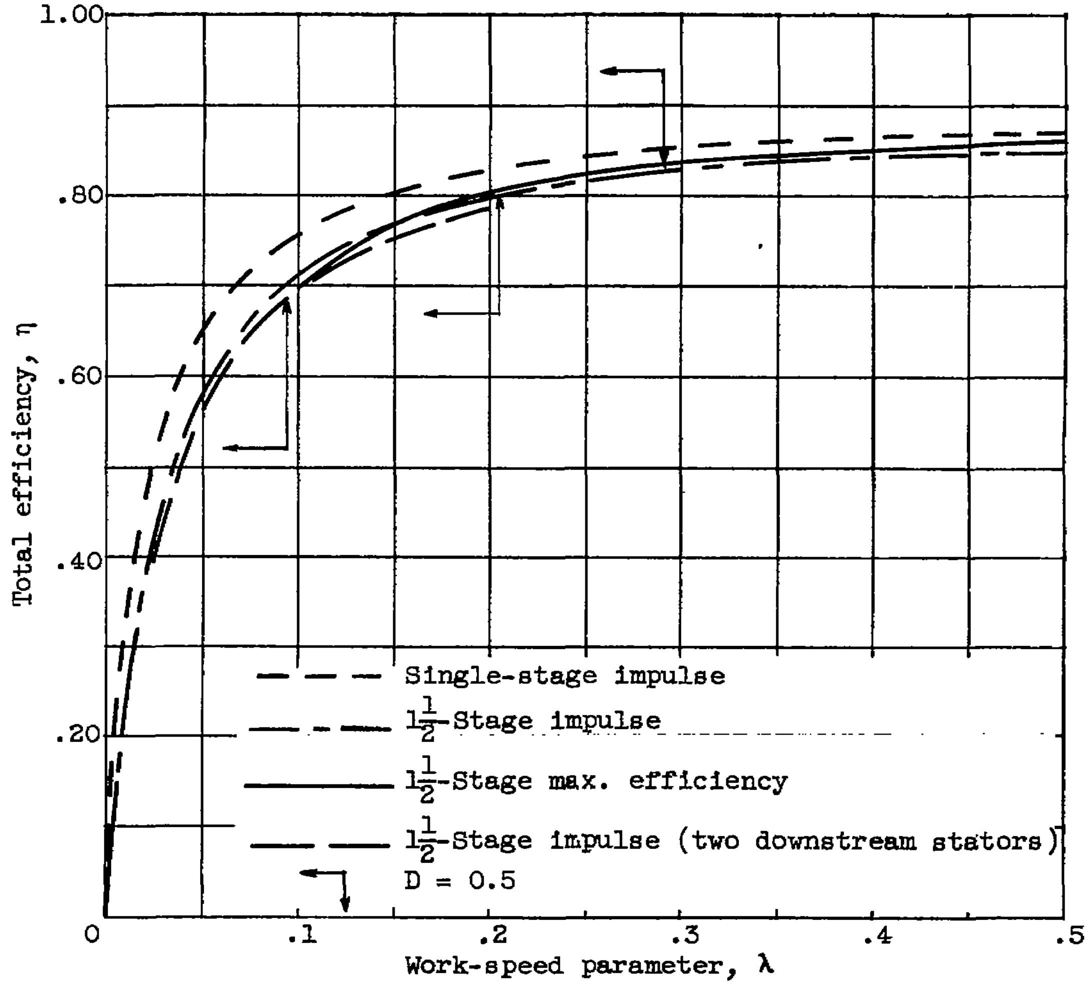


Figure 5. - Comparison of total efficiency of single- and $1\frac{1}{2}$-stage turbines with limiting diffusion factor of 0.5. B, 0.030; $(V_x^2)_{av}/gJ\Delta h'$, 0.49.

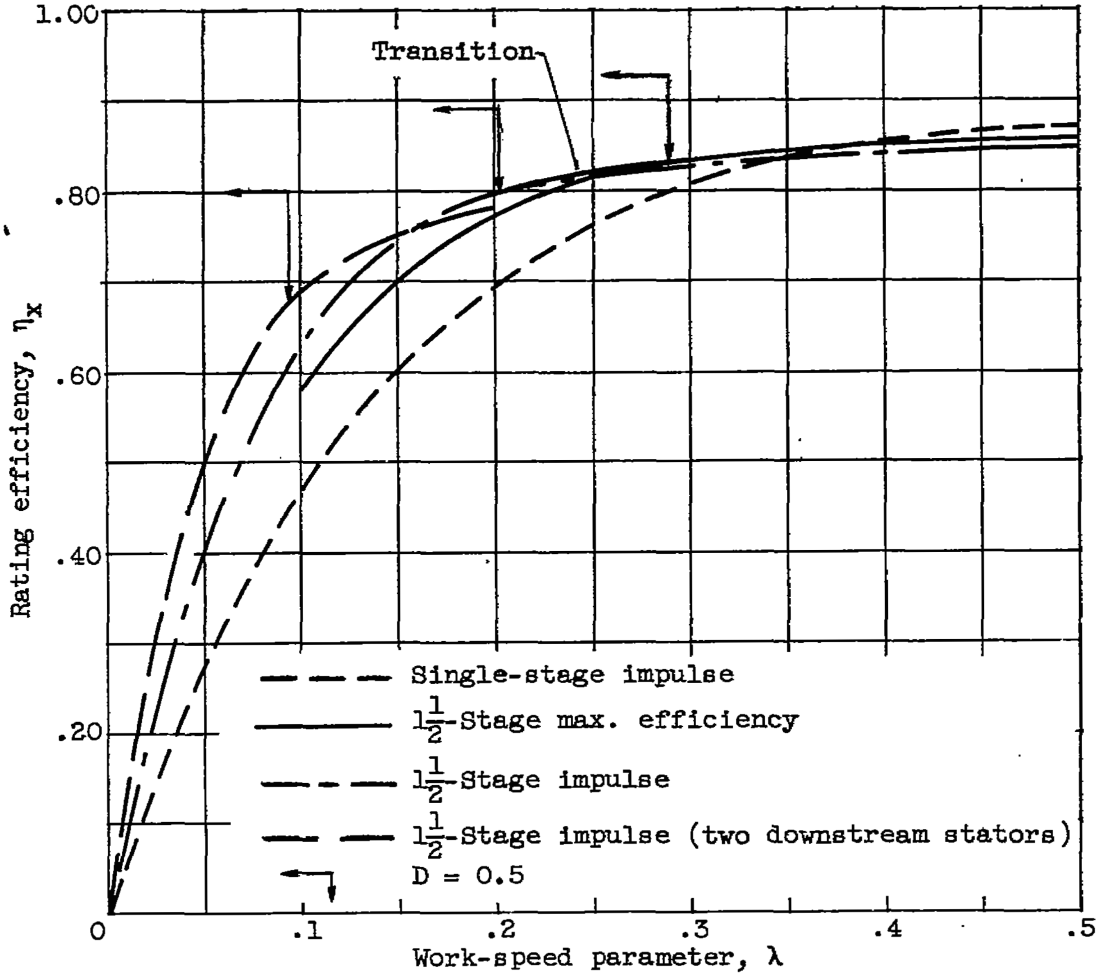


Figure 6. - Comparison of rating efficiency of single- and $1\frac{1}{2}$-stage turbines with limiting diffusion factor of 0.5. B, 0.030; $(V_x^2)_{av}/gJ\Delta h'$, 0.49.

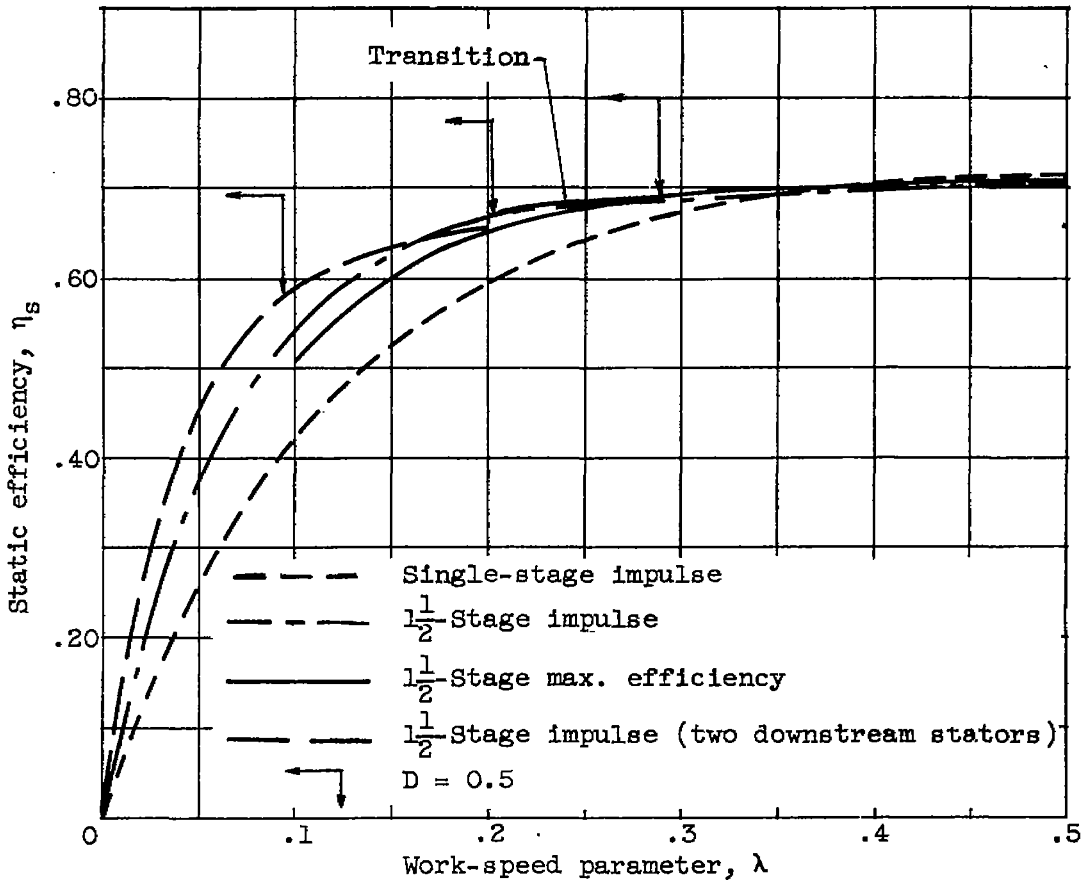
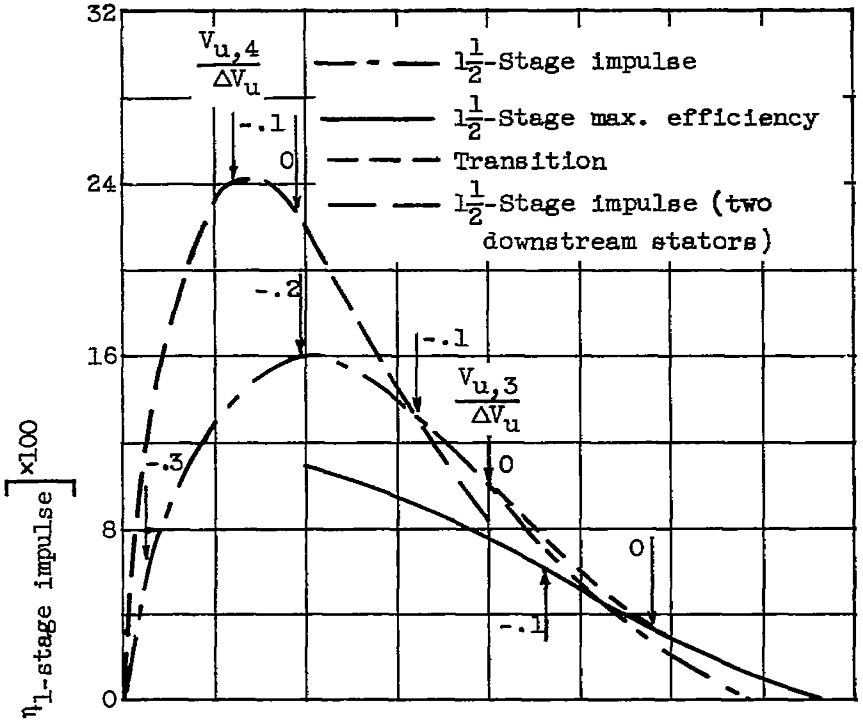
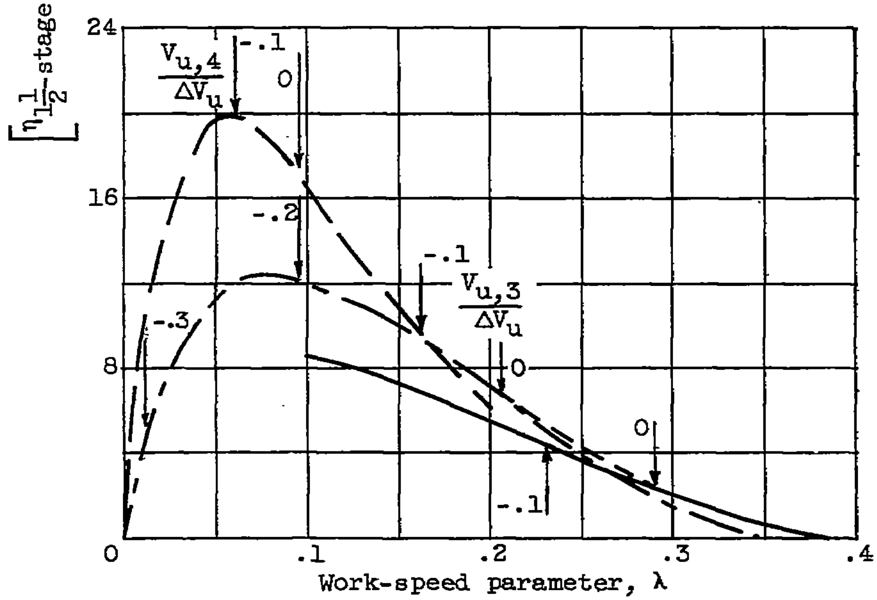


Figure 7. - Comparison of static efficiency of single- and $1\frac{1}{2}$-stage turbines with limiting diffusion factor of 0.5. B, 0.030; $(V_x^2)_{av}/gJ\Delta h'$, 0.49.

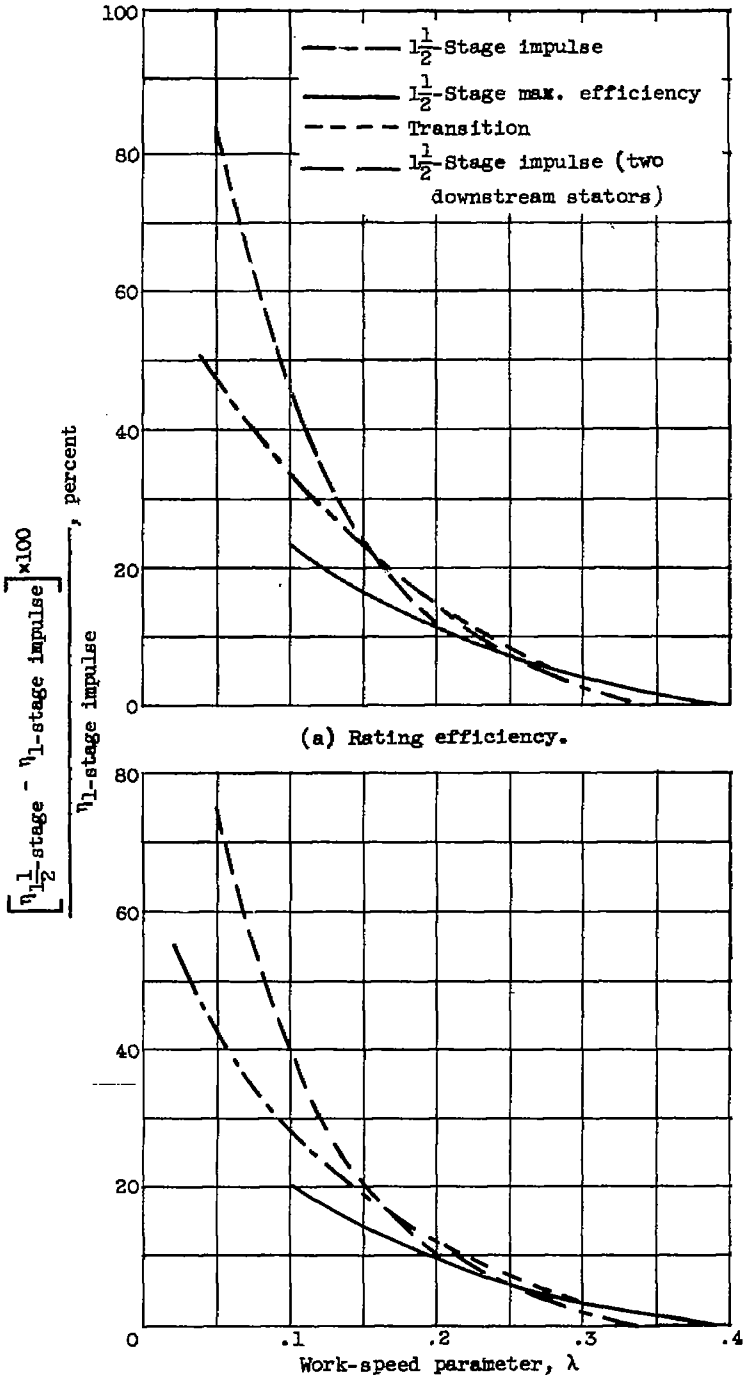


(a) Rating efficiency.



(b) Static efficiency

Figure 8. - Improvements in points of efficiency obtained by use of downstream stators.



(b) Static efficiency.

Figure 9. - Percentage improvements in efficiency obtained by use of downstream stators.

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